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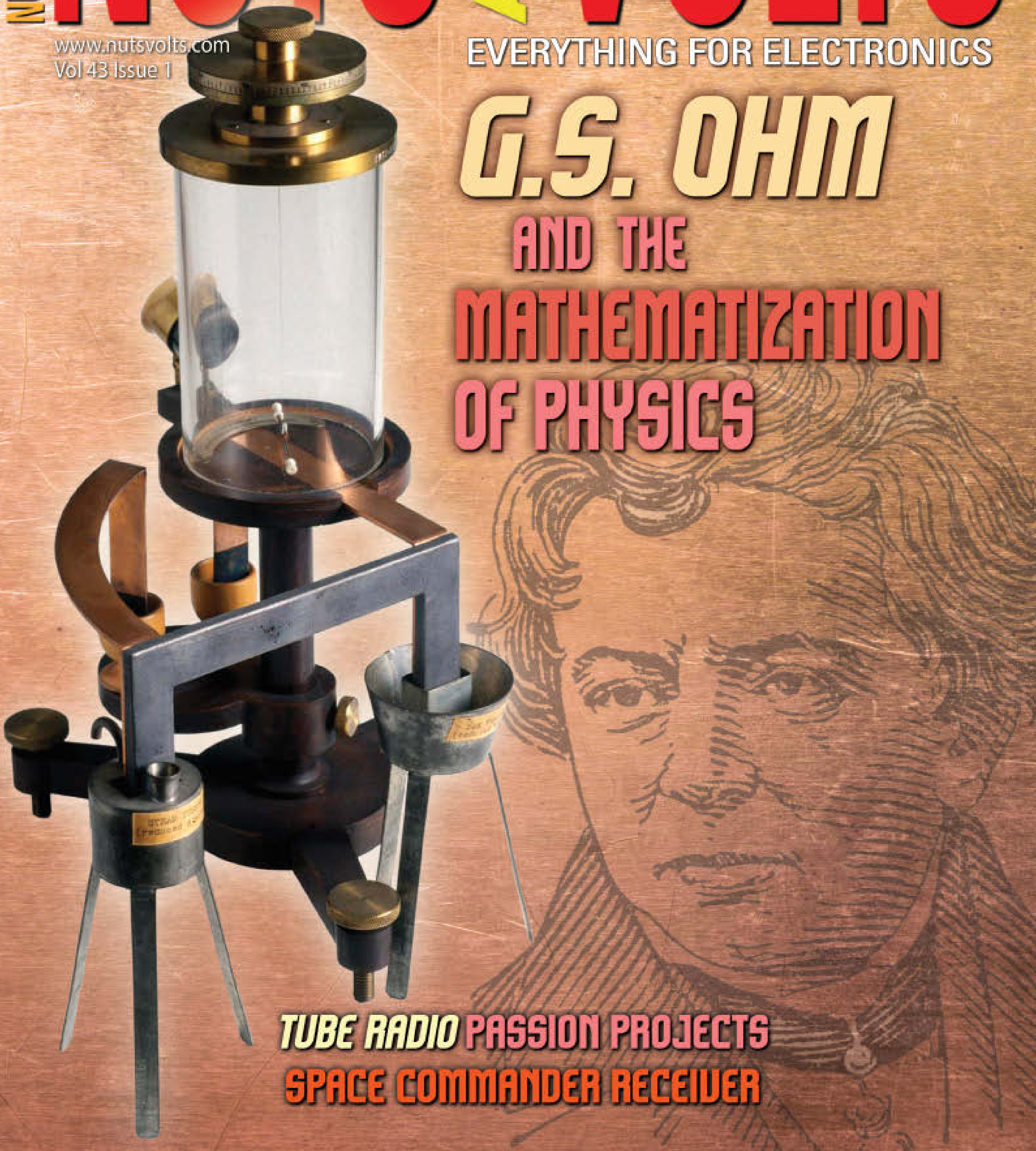

G.S. OHM AND THE MATHEMATIZATION OF PHYSICS

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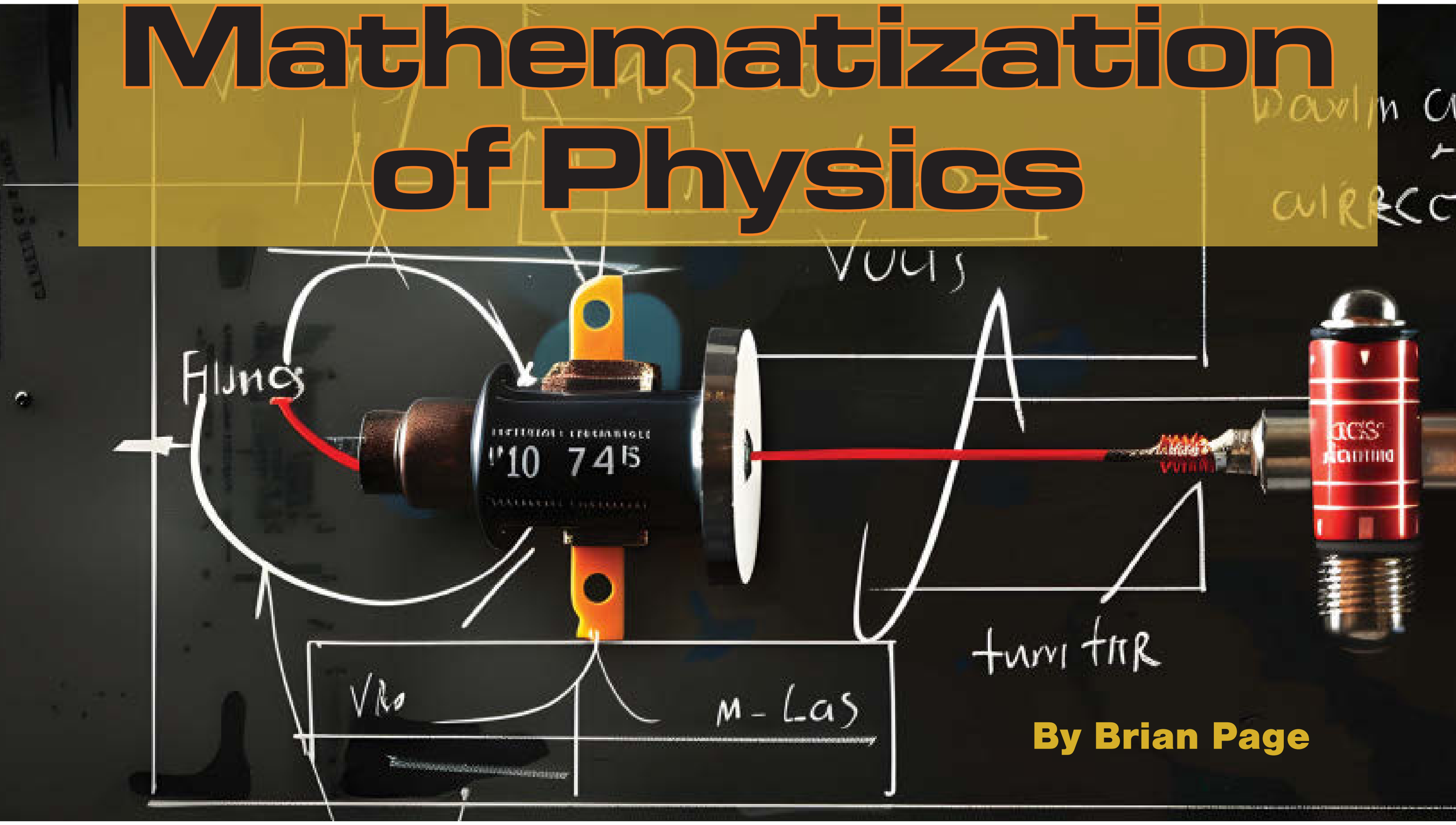
G. S. Ohm and the Mathematization of Physics



By Brian Page

Let's get one misconception out of the way right now. Ohm's Law — defining the relationship between voltage, current, and resistance in an electrical circuit — was not so controversial that Georg Simon Ohm was forced to resign his position as a high school teacher. If the term "urban legend" can be applied to an event that happened nearly 200 years ago, then we in physics have a classic urban legend (**Figure 1**).

The legend that's often repeated and even appears in textbooks from time to time is that when Ohm (1789-1854) published his namesake law, it was so shocking to the simpleton establishment that he was fired from his teaching job and spent years struggling in poverty before foreign scientists finally recognized his genius[1].It's the usual underdog story with the hero pitted against authority. Although this story is the stuff of myth, there is, nevertheless, a kernel of truth in the tale.

That kernel of truth includes educational politics in 19th century Germany, the philosophy of science, and even involves Georg Ohm's brother, mathematician Martin Ohm. It's a reminder that in Ohm's time and even today, advancement and recognition in scientific fields may involve political, philosophical, aesthetic, and personality considerations.

Before digging into the legend, let's first look at Ohm's life and work — and what he did to get himself into trouble.

Alessandro Volta's development in 1800 of the voltaic electrochemical wet cell, plus Hans Christian Ørsted's discovery of electromagnetism in 1820, set the stage for Ohm's first explorations. Ohm was awarded his Ph.D. in 1811, and began teaching secondary school mathematics, a not unusual career path for young academics at the time[2]. Hoping that a striking bit of original research might catapult him out of high school and into a university position, in 1825, Ohm identified electricity as a fertile field in which to make his mark.

To power his experiments, Ohm employed a zinc and copper voltaic pile using dilute sulfuric acid as the electrolyte. To measure current, he created a Coulomb torsion balance that suspended a magnetized needle on a fine wire just above a straight portion of the conductor under test. The needle was first aligned with the earth's magnetic field to lie parallel with the conductor.

With this arrangement, Ohm measured six different lengths of wire ranging from one foot to 75 feet. The current flowing through each conductor caused the torsion balance needle to magnetically deflect by an amount he assumed was proportional to the strength of the current. Ohm charted the amount of deflection for each conductor as a measure in the loss of force (rather than conductance)

When Georg Simon Ohm published his discovery of the relationship between electrical voltage, current, and resistance, his mathematical elucidation of the discovery – in contrast to his experimental announcement – met with skepticism and resistance stemming from differing conceptual frameworks amongst his contemporaries. The differences were not simply a matter of which framework was "right" and which was "wrong." Understanding such conceptional differences remains relevant today in such areas as dark matter research and the unification of relativity with quantum mechanics.

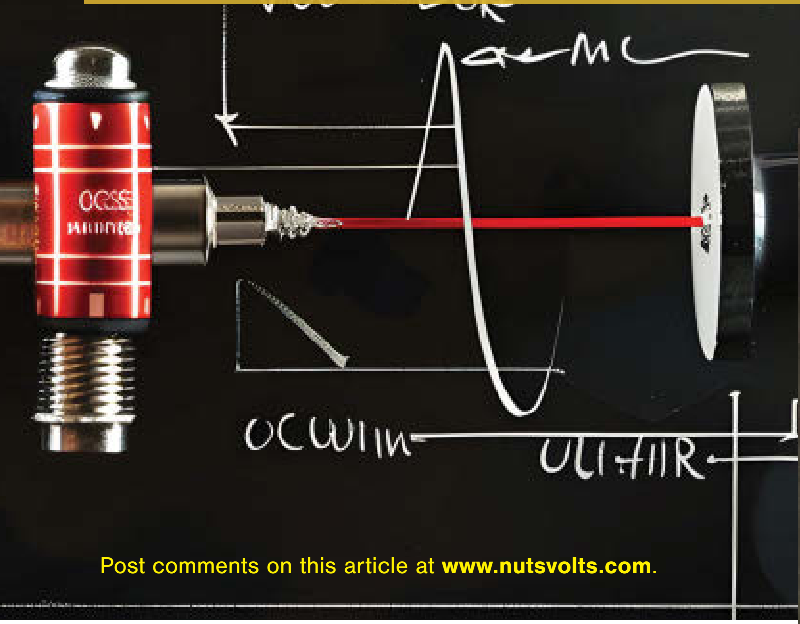


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Figure 1.
A photogravure portrait of Georg Simon Ohm. Ohm devoted his career to advancing the cause of mathematical physics in Germany during the middle years of the 19th century. Although he contributed to the fields of acoustics and crystallography, he is best known for his electrical studies which resulted in Ohm's Law, relating current, voltage, and resistance. ©*The Board of Trustees of the Science Museum (London). Creative Commons BY-NC-SA 4.0.*

compared to a standard, four inch fat copper wire in a metric he called "reduced length or reduced resistance."

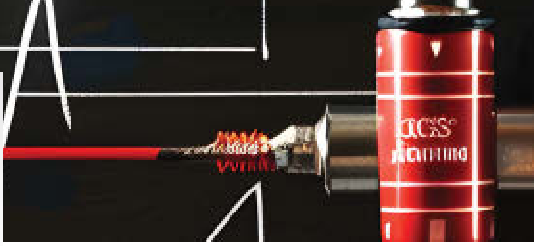
Measurements with the voltaic pile proved troublesome due to polarization of the electrolyte in the wet cell. Polarization results from electrolysis of the electrolyte, with the constituent gases migrating to the poles of the battery caused an unpredictable change in current. To account for this variation, Ohm measured the open circuit voltage (what he referred to as "tension") immediately before and after each conductor test and averaged the results, presuming a linear decline in tension.

With these measurements, Ohm published his results in May 1825 in a paper entitled, "Preliminary Announcement of the Laws, According to which Metals Conduct Contact Electricity." The qualifier "contact" was needed to distinguish a closed-circuit configuration, then known as a galvanic circuit, from static electricity — which to some investigators was thought to be an entirely different kind of electricity. From these data, Ohm discerned a possible relationship described by the formula:

$$v = m \log (x / 1 + a)$$

where:
v = Decrease in force (gauged from magnetic deflection).
m = Coefficient representing the tension (or electromotive force).
x = Length in feet of the conductor under test.
a = Resistance of reference conductor.

Ohm was rather tentative about his coefficient



representing EMF, cautioning that it was "as I have reason to believe ..." and further, "I am at the moment still engaged in making quite sure through more exact experiments of the exact nature of the function[3]."

The paper appeared in *Annalen der Physik und Chemie*, edited by Johann Christian Poggendorff, who in a footnote offered Ohm a suggestion to alleviate the uncertainties introduced by variations in tension produced by the voltaic pile: "It would be wished that the author would find the time to use the so-called thermoelectric circuit to determine these and similar laws. The operations with it are far more steady than those with the so-called hydroelectric circuit and would permit very exact measurements[4]."

Ohm embraced Poggendorff's suggestion with enthusiasm. The device which Poggendorff recommended was Thomas Johann Seebeck's thermocouple. This thermoelectric generator develops a small but steady current when dissimilar metals are in contact and when their opposite surfaces are exposed to a temperature gradient. Ohm's device used copper held at or near freezing in a small basin packed with ice and snow (conveniently available for these January 1826 experiments) and bismuth immersed in boiling water. In contrast to the voltaic pile which had "a very damaging influence," Ohm found the thermoelectric voltage to remain stable for 30 minutes[5]. Refer to **Figure 2**.

Figure 2.
A reproduction of Ohm's experimental apparatus. The thermoelectric generator in the foreground used a copper and bismuth thermocouple activated with boiling water and snow. The torsion balance consisted of a magnetized needled positioned directly over and parallel to a copper strip connected to a test conductor, the ends of which attached via the two small mercury-filled cups in the far background. *©The Board of Trustees of the Science Museum (London). Creative Commons BY-NC-SA 4.0.*

With the new steady source, Ohm repeated his conductivity tests, and he had good reason to do so. After publishing his 1825 report, he received accounts of similar experiments by Peter Barlow and Henri Becquerel, all with differing results. Ohm concluded "that neither the laws given by these two investigators on the length of the conductor, nor that announced by me, are general and free from interference by forces which in no way concern the matter in question[6]."

In this set of simple but brilliant experiments, Ohm tested eight copper wires of 2, 4, 6, 10, 18, 34, 66, and 130 inches in length. In these more carefully controlled tests, Ohm reached a different conclusion than his previous estimation. He now expressed the relationship between tension (voltage) and magnetic deflection (current) with the formula:

$$X = a / (b + x)$$

where:
X = Torsion balance needle deflection (current).
a = Tension (EMF) of the source taken as the thermocouple temperature gradient.
b = A constant representing the internal resistance of the source.
x = Reduced length (resistance) of the conductor under test.

Ohm repeated these experiments several times over several days to minimize error variations due to temperature differences and component contact resistances, tabulating the results. He then substituted brass wire for copper and found that brass obeyed the same relationship.

Finally, and critically, Ohm varied the thermocouple temperature gradient in a test that clinched his discovery: "First, the fact is worthy of note that the value of b remains unaltered while the force [tension or voltage] is more than ten times smaller, so that a appears to depend simply on the exciting force, b simply on the unchanged part of the conducting circuit. Secondly, it appears to follow from these experiments that the force in the thermoelectric circuit is exactly proportional to the temperature difference between its two points of excitation[7]."

Additionally, Ohm confirmed Humphry Davy's findings that resistance varied with the temperature of the conductor under test. He measured lower resistance with snow packed around the conductor, and higher resistance when it was heated with a spirit lamp. To Ohm, this may have indicated an affinity between electricity and heat conductivity as elucidated by Joseph Fourier.

Finally, Ohm discerned one other relationship that had far-reaching consequences. He determined that "Cylindrical conductors of the same substance but different diameter, have the same conductivity values if their lengths are in proportion to their cross-sections[8]." This finding implies that the electrical current moves through the entire cross-section of the conductor and not along the surface as static

electricity was known to do.

Ohm concluded his February 1826 published account singing the praises of Seebeck's thermoelectric source that made the discoveries possible: "Effects of the galvanic circuit, to all appearance [heretofore] widely different, sort themselves out in all their variety into a beautiful whole. Seebeck's important discovery appears to spin the thread which leads us out of the labyrinth into which the electric current has grown[9]."

He clearly grasped that he had brought a measure of order to the massive confusion and speculation surrounding the nature of electricity, giving the first precise definitions to what we call current, voltage, and resistance. Still, although Ohm had established the proportionality of voltage to resistance, he had not quite taken that final step in establishing our modern Ohm's Law equation. He took that step in his next paper.

Ohm wasted little time further analyzing his experimental results. In April 1826, he published "Attempt of a theory of the electroscopic phenomena produced by galvanic forces." In this analysis, he first elaborated and then simplified his discovery:

$$S = \kappa \omega (a / l)$$

where:
S = Magnetic deflection (current).
κ = Conductibility of the test conductor.
ω = Cross-section of the test conductor.
a = Tension (voltage).
l = Length of the test conductor.

Ohm then consolidated the three physical properties of the conductor, κ, ω, and l, into a single variable (reduced length), signifying total resistance, resulting in the Ohm's Law for which he is immortalized:

$$S = a / l$$

where:
S = Magnetic deflection (current).
a = Tension (voltage).
l = Teduced length (resistance) of test conductor.

This second paper marks the true discovery and announcement of Ohm's Law and, together with the first paper of 1826, represents an ideal example of experimental method and data analysis. Nevertheless, Ohm was not satisfied with simply discovering new phenomena. He was an early proponent of theoretical physics in Germany and wished to transform the way physics was taught and practiced, adopting the methods that physics was then being practiced in France. And he had not yet exhausted mathematical analysis of electrical current flow.

To dig deeper, Ohm needed time to fully flesh out his discovery. At his brother Martin's suggestion, Georg requested and was granted a one year leave of absence at half-pay from his teaching position. He was not fired. Ohm's research had been recognized as fruitful by the education authorities and deemed worthy of further elaboration. It was a validation of his abilities.

With that, Ohm packed up and moved from Cologne in the western region of Germany to Berlin — the intellectual capital of the German States[10] — on the opposite side of the country, moving in with his brother, who was a professor of mathematics at the Military College of Berlin.

Ohm used his sabbatical to produce his masterpiece, a book entitled, *The Galvanic Circuit Investigated Mathematically* (in German: *Die galvanische Kette, mathematisch bearbeitet*), intended as a complete and definitive exploration of electrical current.

Let's look first at the contents and then consider Ohm's approach and presentation that caused difficulties with other investigators and led to his confusing legacy that remains even today.

The book, published in 1827, consists of three roughly equal-length sections: Introduction; The Voltaic Circuit; and Appendix. Ohm opens with a statement of intent: "The design of this Memoir is to deduce strictly from a few principles, obtained chiefly by experiment, the rationale of those electrical phenomena which are produced by the mutual contact of two or more bodies, and which have been termed galvanic ...[11]"

Although Ohm specifically cites the experimental foundation of his work, nowhere does he detail any of it. Next, Ohm lists the three "fundamental laws" upon which his work was based:

1. Electricity passes only between adjacent particles in an amount equal to their difference in potential.
2. Loss of electricity to the air is proportional to the electroscopic force (voltage), the exposed surface area, and a coefficient characteristic of the air.
3. Two bodies in contact possess a difference in electroscopic force at their common surface.

Ohm uses three geometrical figures — indeed, the only illustrations in the entire manuscript — to graphically display circuit configurations, first with tension diminishing linearly with increasing resistance, and then more elaborate circuits having parallel paths and conductor portions varying in resistance. The circuit element permutations are exhaustive, concluding with "A Circuit Conductor formed of Two Parts which are neither the Same Size or of the Same Material[12]."

Following the geometrical explanations, Ohm then restates in greater detail the substance of this most recent journal paper without, notably, providing a description of the experiments or any of the raw data. He then demonstrates the predictive power of his discovery, explaining the resistance and current characteristics of the voltaic pile, Seebeck's thermoelectric apparatus, and galvanometers.

This first section of the book is often described as a geometrical explanation for those readers unable to follow the mathematical treatment in the second part. That's not really true. While the Introduction requires only facility with geometry and algebra, the two sections differ completely in subject matter. The first 83 pages may be thought of as illustrating the practical implications and applications of Ohm's Law. The next section goes much deeper, as Ohm saw it, into the underlying physics.

Recall the first and third of the "fundamental laws" upon which Ohm's treatise was based. These principles inspired Ohm in the second section to model the flow of electricity in a conductor after the manner Joseph Fourier modeled heat flow in his 1822 publication, *The Analytic Theory of Heat*. For Ohm, the resemblance was more than simply analogous: "The form and treatment of the differential equations thus obtained are so similar to those given for the propagation of heat by Fourier and Poisson, that even if there existed no other reasons, we might with perfect justice draw the conclusion that there exists an intimate connection between these natural phenomena; and this similarity increases as we continue to pursue the subject[13]."

The "other reasons" included the fact that Seebeck's thermoelectric device generates tension via heat differential. In Fourier's work, the independent variable was the temperature gradient; for Ohm, it was tension or electromotive force. In the same manner that Fourier reduced the conduction of heat to these three attributes:

1. Conducting area.
2. Temperature gradient.
3. Thermal conductivity of the material.

Ohm's presentation mirrored Fourier's attributes almost precisely:

1. Conductor length and diameter.
2. Tension (voltage) gradient.
3. Conductivity of the material.

Ohm based his analysis on his January 1826 discovery of the proportionality of conductor length and diameter, that current in a galvanic circuit moves through the entire cross-section of the conductor, and his postulate that "each particle of the conducting medium situated in the circuit of action receives each moment just the same amount of transmitted electricity from the one side as it gives off to the other[14]." Thus, he modeled the conductor as a contiguous set of infinitely thin disks, applying Taylor's theorem and boundary (tension) values, eventually arriving once again at Ohm's Law:

$$S = A / L$$

where:
S = Magnetic deflection (current).
A = Tension (voltage).
L = Reduced length (resistance) of test conductor.

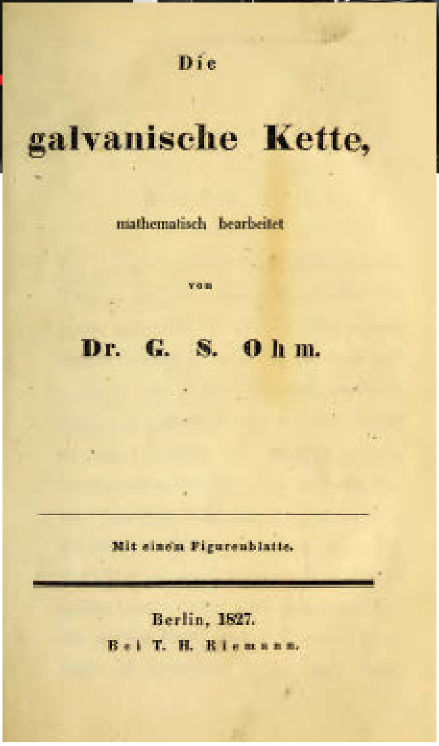
In the final portion (Appendix), Ohm applied his analysis to circuits that result in chemical changes in fluids.

It's important to consider Ohm's philosophical approach to physics indicated by his 1827 book. He was not content with merely describing the empirical results of his careful experiments. For Ohm, the analytical treatment was essential for establishing the truth of what his experiments had indicated, hinted at. In a sense, the journal articles of 1826 were, to Ohm, incomplete, perhaps lacking foundation, without also the rigorous mathematical understanding embodied in *The Galvanic Circuit Investigated Mathematically*. He never assigned specific physical meaning to the phenomena that he measured.

Ohm undertook his sabbatical in hopes of advancing his career. To that end, he chose to resign from his position in Cologne (where Peter Gustav Lejeune Dirichlet had been his student) and remain in Berlin with expectations of greater opportunities to come. Unfortunately, it didn't work out like that, at least in the short term.

The fundamental essence of Ohm's research, what we call Ohm's Law, was readily adopted by his contemporaries, specifically men such as Gustav Fechner (who took pains to confirm Ohm's work), Moritz von Jacobi, Emil Lenz, Wilhelm Eduard Weber, Franz Ernst Neumann, and Carl Friedrich Gauss — all names familiar to physicists today. In other words, there was nothing scandalous about current's, resistance's, and voltage's ménage à trois relationship. Nevertheless, there was criticism of Ohm's efforts, some of it deserving, some political, and the most damaging, philosophical.

The most perceptive criticism of Ohm's work came from Georg Friedrich Pohl, a professor of physics in Berlin, and rested on both technical and conceptual differences. It had long been known that the difference in potential of a power source measured in an open circuit disappears when that circuit is closed with a load connected (the view of André-Marie Ampère). For Ohm to suggest that an EMF exists between adjacent particles in the closed circuit indicated to Pohl that Ohm lacked a fundamental understanding of electrical phenomena, writing that Ohm possessed, “the most innumerable confusions in physics[15].” Thus, Ohm’s detailed analysis in the second section of his book was viewed as a pleasant exercise in mathematics but utterly without relevance to the phenomena.



Die

galvanische Kette,

mathematisch bearbeitet

von

Dr. G. S. Ohm.

Mit einem Figurenblatte.

Berlin, 1827.
Bei T. H. Riemann.

Figure 3.
The title page of Ohm’s 1827 book, *The Galvanic Circuit Investigated Mathematically*. The qualifier “Mathematically” emphasizes Ohm’s focus on the theoretical analysis of the electrical circuit pointedly in contrast to a presentation of mere experimental work.

Additionally, Ohm’s modeling of current flow on Fourier’s analysis of heat flow was — and is — imperfect. Heat conductors store heat once the temperature gradient is removed, whereas electrical conductors do not remain charged once the circuit is opened. Fortunately, this does not impact the steady-state equations which served as a model for Ohm, but it could lead a reviewer to conclude that the book was, “the result of an incurable delusion[16].”

Technical objections such as these could be answered technically, and with further demonstration via experiments. Other objections were not so easily countered.

In the political realm, Georg’s brother Martin had stepped on toes in the German ministry of education for promoting improvements in mathematical education, specifically recommending a geometry text that Georg had published in 1817, which made Georg guilty by association.

At the time of Ohm’s publication, the older generation of physicists in Germany tended to hold a different philosophical and conceptual view of physics, more experimental and hostile to deeper analysis. Ohm’s tenet that, “Every theory of a class of natural phenomena founded upon facts which will not admit of analytic investigation in the form of its exposition is imperfect ...[17]” could be taken as a shot across the bow for such critics as C. H. Pfaff who countered that, “A physical explanation penetrates further than a so-to-speak mathematical explanation, which gives only a formula for the quantitative determination of the phenomena[18].”

Whereas Ohm never speculated on electrical fluids, G. W. Muncke could lament that, “... one cannot for a moment fail to recognize that we now require observations and experiments much more than calculations and geometrical formulas[19].” These were not quibbles. Ohm and his critics operated in different conceptual frameworks that were unlikely to be reconciled.

In explaining why Ohm omitted the details of his experiments from the 1827 publication, some historians speculate that perhaps Ohm believed his early journal articles were more well-known than they were in fact. I think such an interpretation does a disservice to Ohm. From the text of *The Galvanic Circuit Investigated Mathematically*, it’s clear that Ohm knew exactly what he was doing and that had he included the experimental aspects of his research program; the focus would be on his results and not on the abstracted mathematical foundations. The major finding of his work — Ohm’s Law — is hardly emphasized in the text. Instead, the book is a guided example of mathematical analysis using the properties of the electrical circuit as an example. His goal is stated quite plainly in the title of the book: *Investigated Mathematically*. Ohm advocated a new way to do physics (**Figure 3**).

So, was G. S. Ohm responsible for the mathematization of physics? Not exactly. At the time Ohm published, France was the center of gravity for mathematical physics. After all, Ohm’s hero, Jean-Baptiste Joseph Fourier was French, as was Pierre-Simon Laplace — two giants in the field. Still, Ohm had influence. His 1827 publication was the most sophisticated mathematical physics to appear in Germany, and it began a tradition that helped move that center of gravity from France to Germany for the latter half of the 19th century.

How does the controversy surrounding Ohm and his law impact us today? Consider that in our conceptual framework, we talk about subatomic entities as “particles” and “waves” although we “know” that they are neither. We may speak of an electron, for instance, as a particle with wave-like properties, knowing that it’s now best to consider it simply as a mathematical construct, a wave function.

This dilemma of how we represent such concepts mirrors the reception of Ohm’s early attempt to mathematize physics. Those resisting Ohm’s Law did so, not because they objected to the equation relating current, voltage, and resistance. They objected that Ohm had not actually explained anything; that he had merely described theoretical phenomena without advancing knowledge of the mysterious electrical force that somehow flows through conductive wires.

It’s as if an acquaintance demands to know whether

an electron is a particle or a wave and you reply that it's a probability amplitude, and its modulus square gives the probability density of position. Your response is correct, but your friend may feel that this is no answer at all. The question was seated in one conceptual framework, and your reply was based on a different conceptual framework. In the same way, Ohm's critics were similarly dissatisfied.

But might these critics have a point? The use of terms such as particle, wave, field, force, and spin are models which we use to our convenience in a conceptual framework describing physical phenomena, acknowledging that a model is merely an approximation of a postulated reality. While we often consider the mathematical expression to be superior to a physical framework, in the end the mathematical analysis is just another model. The difference is in degree.

As theoretical physicist John Archibald Wheeler famously noted, "... every law of physics, pushed to the extreme, will be found to be statistical and approximate, not mathematically perfect and precise[20]." When a physicist such as Ohm reduces a phenomenon to an abstract mathematical expression, has that phenomenon been fully explained?

The reception accorded to Ohm's book wasn't simply a case of simpletons unable to understand a mathematical treatment. Rather, it was a first stumbling encounter with what Eugene Wigner termed "The Unreasonable Effectiveness of Mathematics in the Natural Sciences," where he writes, "... we cannot know whether a theory formulated in terms of mathematical concepts is uniquely appropriate[21]." Are the mathematical expressions more "real" than the phenomena? Or are the statistical results of experimental physics truer to nature than theoretical physics?

It may be giving Ohm's critics more credit than they deserve, but ultimately their questioning of a purely mathematical description of nature turns out to have relevance into the 20th century and our time as well with, for example, attempts to unify General Relativity and Quantum Mechanics.

Does the failure of such unification attempts point to limitations in our mathematical models of these phenomena? Another example is the hunt for dark matter. The mathematical necessity for the substance is airtight. Yet, experimental verification to date has not only failed detection, but has steadily circumscribed the possibility of its existence.

In 1849, Ohm finally achieved his goal of a university professorship. He died five years later. However, G. S. Ohm remains relevant to this day. **NV**

Footnotes

[1] Two examples are: Antony Anderson, "Spare a Thought for the Ohm," *New Scientist*, 114 (7 May 1987), p. 64. And Elizabeth C. Patterson, "Why Ohm?" *American Scientist*, 77 (May/June 1989), p. 309.

[2] Christa Jungnickel and Russell McCormmach, *Intellectual Mastery of Nature: Theoretical Physics from Ohm to Einstein, Volume 1, The Torch of Mathematics*, 1800 – 1870 (Chicago: University of Chicago Press, 1986).

[3] Sanford P. Bordeau, *Volts to Hertz: The Rise of Electricity* (Minneapolis: Burgess Publishing Company, 1982), p. 91.

[4] Joseph F. Keithley, *The Story of Electrical and Magnetic Measurements: From 500 BC to the 1940s* (New York: IEEE Press, 1999), p. 95.

[5] G. S. Ohm, "Determination of the Law in Accordance with which Metals Conduct Contact Electricity Together with an Outline of a Theory of the Voltaic Apparatus and of Schweigger's Multiplier," *Journal für Chemie und Physik*, 48 (1826), translated by N. H. de Vaudrey Heathcote, *Science Progress*, 26:101 (July 1931), p. 64.

[6] Ohm (1826)

[7] Ohm (1826)

[8] Ohm (1826)

[9] Ohm (1826)

[10] At the time, Germany was a loose confederation of 39 nominally independent states, four of which were monarchies. It is more accurate to refer to this area of Europe as the German States.

[11] G. S. Ohm, *Die galvanische Kette, mathematisch bearbeitet* (Berlin: T. H. Rieman, 1827), translated by William Francis, *The Galvanic Circuit Investigated Mathematically* (New York: D. Van Nostrand Company, 1891), p. 11.

[12] Ohm (1827), p.35.

[13] Ohm (1827), p. 15.

[14] Ohm (1827), p. 21.

[15] Morton L Schagrin, "Resistance to Ohm's Law," *American Journal of Physics*, 31:7 (July 1963), p. 546.

[16] H. J. J. Winter, "The Reception of Ohm's Electrical Researches by his Contemporaries," *Philosophical Magazine*, 35 (1944), p. 373.

[17] Ohm (1827), p. 110.

[18] Kenneth Caneva, "From Galvanism to Electrodynamics: The Transformation of German Physics and Its Social Context," *Historical Studies in the Physical Sciences*, 9 (1978), p. 16. I am indebted to Professor Caneva for untangling the complex social environment of early 19th century German science.

[19] *Caneva* (1978), p. 88.

[20] John Archibald Wheeler and Wojciech Hubert Zurek, *Quantum Theory and Measurement* (Princeton: Princeton University Press, 2016), p. 210.

[21] Eugene Wigner, "The Unreasonable Effectiveness of Mathematics in the Natural Sciences," *Communications in Pure and Applied Mathematics*, 13:1 (February 1960); reprinted: Douglas M. Campbell and John C. Higgins, *Mathematics: People, Problems, Results* (Belmont, CA: Wadsworth, Inc., 1984), p. 117. See also: David Lindley, *The Dream Universe: How Fundamental Physics Lost Its Way* (New York: Doubleday, 2020).